TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 3 2016

Calculator Assumed

Geometric proofs, vector proofs, relative motion

STUDENT'S NAME ______________________________

DATE: **TIME:** 50 minutes **MARKS**: 50

INSTRUCTIONS:

Standard Items: Pens, pencils, ruler, eraser.

Special Items: Three calculators, drawing instruments, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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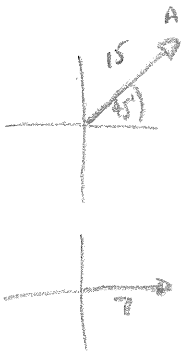
1. (4 marks)

Ship A is sailing north-east at 15 km per hour. To an observer on ship A, ship B appears to be moving east at 7 km per hour. Calculate the actual magnitude and direction of ship B.

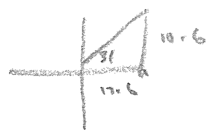


$\underset{\sim}{V_A} = \begin{pmatrix} 15\cos 45° \\ 15\sin 45° \end{pmatrix}$ ✓

${}_B\underset{\sim}{V}_A = \begin{pmatrix} 7 \\ 0 \end{pmatrix}$ ✓

$\therefore \underset{\sim}{V_B} = \begin{pmatrix} 7 \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{15}{\sqrt{2}} \\ \frac{15}{\sqrt{2}} \end{pmatrix}$

$= \begin{pmatrix} 17.6 \\ 10.6 \end{pmatrix}$ ✓



$|\underset{\sim}{V_B}| = 20.55 \text{ km h}^{-1}$ ✓

direction $= 059°$

2. (4 marks)

Given ${}_{A}\boldsymbol{r}_{B} = \begin{pmatrix} 2 \\ 10 \end{pmatrix}$, ${}_{B}\boldsymbol{r}_{C} = \begin{pmatrix} -11 \\ 9 \end{pmatrix}$ and $\boldsymbol{r}_{C} = \begin{pmatrix} 8 \\ 2 \end{pmatrix}$. Determine $\boldsymbol{r}_{A}$

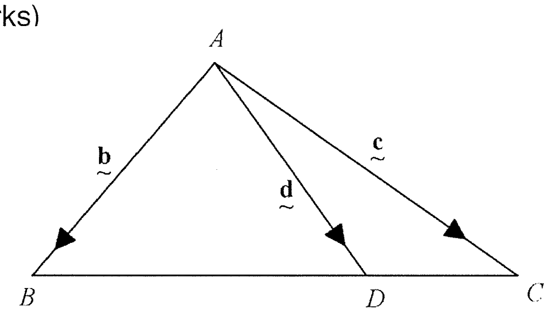
$\begin{pmatrix} 2 \\ 10 \end{pmatrix} = \underset{\sim}{r}_A - \underset{\sim}{r}_B$ ✓

$\begin{pmatrix} -11 \\ 9 \end{pmatrix} = \underset{\sim}{r}_B - \underset{\sim}{r}_C$ ✓

adding gives $\begin{pmatrix} -9 \\ 19 \end{pmatrix} = \underset{\sim}{r}_A - \begin{pmatrix} 8 \\ 2 \end{pmatrix}$ ✓

$\therefore \underset{\sim}{r}_A = \begin{pmatrix} -1 \\ 21 \end{pmatrix}$ ✓

3. (~~3~~ 4 marks)



Given that $\overrightarrow{BD} = 2\overrightarrow{DC}$, show that $\underset{\sim}{\mathbf{b}} + 2\underset{\sim}{\mathbf{c}} = 3\underset{\sim}{\mathbf{d}}$.

$\overrightarrow{BD} = \underset{\sim}{d} - \underset{\sim}{b}$ ✓

$\overrightarrow{DC} = \underset{\sim}{c} - \underset{\sim}{d}$ ✓

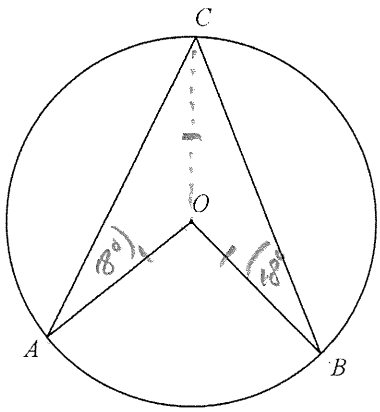
now $\underset{\sim}{d} - \underset{\sim}{b} = 2(\underset{\sim}{c} - \underset{\sim}{d})$ ✓

$= 2\underset{\sim}{c} - 2\underset{\sim}{d}$

$\therefore 3\underset{\sim}{d} = \underset{\sim}{b} + 2\underset{\sim}{c}$ ✓

4. (~~4~~ 3 marks)

The diagram shows a circle with centre O. Given that $\angle CAO = 18^\circ$ and $\angle CBO = 18^\circ$. Determine the size of $\angle AOB$.



join OC ✓

∴ ∠'s OCB + OCA = 18° ✓ (isosceles Δ = radii)

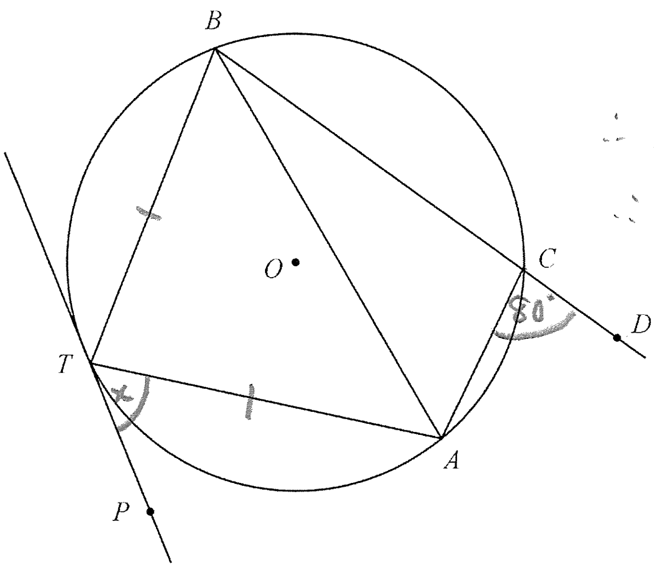
∴ ∠'s COB + COA = 144° ✓ (angle Σ Δ)

∴ ∠AOB = 360 − 2(144) ✓

= 72°

5. (4 marks)

In the diagram below PT is a tangent at T. TB = TA and $\angle DCA = 80^\circ$. Determine the size of $\angle PTA$.



∠BCA = 100° (angles on a st line) ✓

∴ ∠BTA = 80° (cyclic quad) ✓

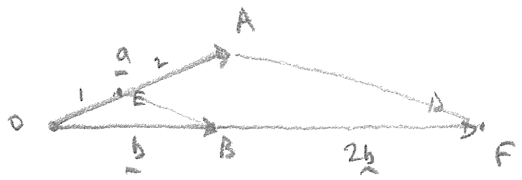
∴ ∠TBA = 50° (isosceles Δ) ✓

∴ ∠PTA = 50° (alt seg thm) ✓

6. (7 marks)

$\overrightarrow{OA} = \underset{\sim}{\mathbf{a}}$ and $\overrightarrow{OB} = \underset{\sim}{\mathbf{b}}$. E is the point on OA such that OE : EA = 1 : 2. F is the point such that $\overrightarrow{BF} = 2\underset{\sim}{\mathbf{b}}$.

(a) Express in terms of $\underset{\sim}{\mathbf{a}}$ and $\underset{\sim}{\mathbf{b}}$, $\overrightarrow{OE}, \overrightarrow{EB}, \overrightarrow{OF}$ and $\overrightarrow{AF}$. [4]



$\overrightarrow{OE} = \frac{1}{3}\underset{\sim}{a}$ ✓

$\overrightarrow{EB} = \underset{\sim}{b} - \frac{1}{3}\underset{\sim}{a}$ ✓

$\overrightarrow{OF} = 3\underset{\sim}{b}$ ✓

$\overrightarrow{AF} = 3\underset{\sim}{b} - \underset{\sim}{a}$ ✓

(b) Show that EB is parallel to AF. [2]

$\overrightarrow{EB} = \frac{1}{3}(3\underset{\sim}{b} - \underset{\sim}{a})$ which is $k\overrightarrow{AF}$

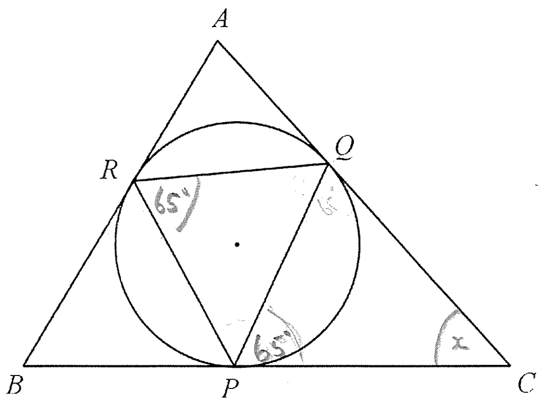
$\therefore$ //

(c) Determine the ratio of the lengths EB : AF [1]

1 : 3

7. (4 marks)

The circle in the diagram touches the triangle ABC at P, Q and R. ~~∠BRP = 61°~~, ~~∠RPQ = 54°~~, and ∠PRQ = 65° Determine the size of ∠ACB.



∠QPC = 65° (alt seg thm) ✓

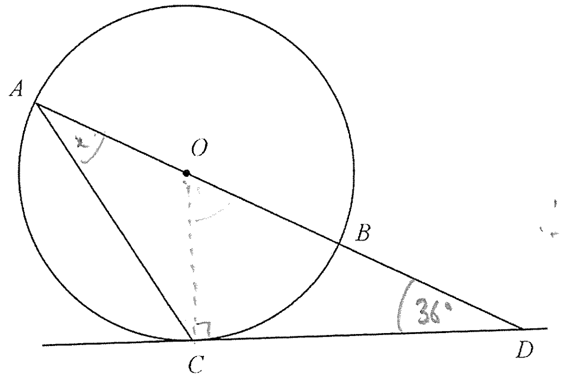
∠PQC = 65° (Δ PQC isosceles = tangents) ✓

∴ ∠ACB = 180° − (65° + 65°) ✓

= 50° (angle sum of Δ) ✓

8. (4 marks)

The diameter AOB of the circle below is produced to meet the tangent CD at D. Given that ∠ADC = 36°. Calculate the size of ∠DAC.



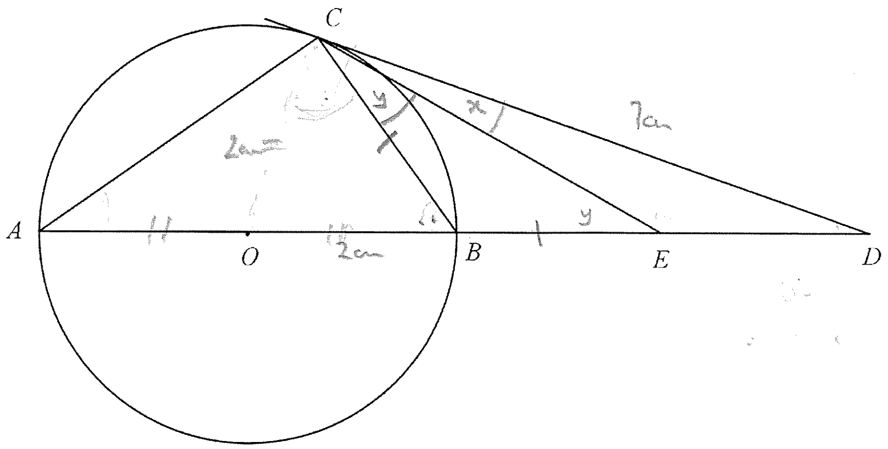
join OC

∠DOC = 54° (angle sum of Δ)

∴ ∠DAC = 27° (angle at centre = 2x angle at circ)

8. (9 marks)

Triangle ABC is inscribed in a circle with AB as a diameter. The tangent at C meets AB produced at D, the point E is on the line BD such that BE = BC. Given that $\angle DCE = x^\circ$ and $\angle BCE = y^\circ$.



(a) Calculate, in terms of x and y only, the angles CEB, CBA and CAB. [3]

$\angle CEB = y^\circ$ (isosceles Δ)

$\angle CBA = 180^\circ - \angle CBE$

$= 180^\circ - (180 - 2y) = 2y^\circ$

$\angle CAB = 90 - 2y^\circ$ (since $\angle ACB = 90$ (angles in a semicircle))

(b) Write an equation for y in terms of x. [3]

$\angle OCB = 2y^\circ$ (Δ OCB isosceles = radii)

$\therefore x + y + 2y = 90^\circ$ (DC a tangent - tangent-radius)

$\therefore 3y = 90 - x$

$y = 30 - \frac{x}{3}$

[OR $\angle BCD = \angle CAB$ (alt seg thm)

$\therefore x + y = 90 - 2y$

$3y = 90 - x$

$y = 30 - \frac{x}{3}$]

(c) If the length of DC = 7 cm and the radius of the circle is 2 cm, show that DB (z) is given by $z^2 + 4z - 49 = 0$. [3]

DB = z $\therefore$ in Δ OCD $\angle OCD = 90^\circ$ (tangent)

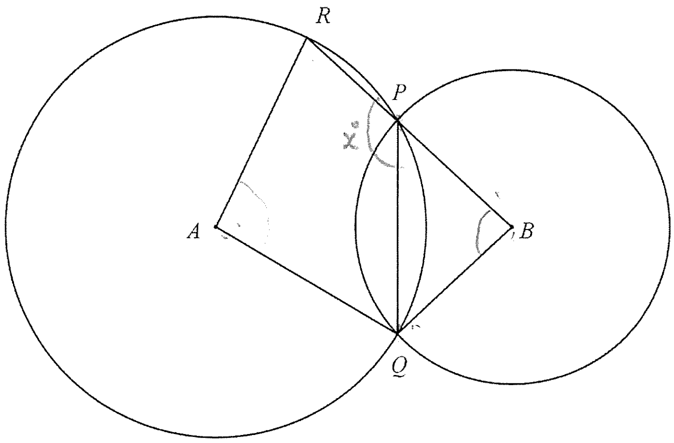
ie $(z+2)^2 = 2^2 + 7^2$

$z^2 + 4z + 4 = 4 + 49$

$z^2 + 4z - 49 = 0$

9. (7 marks)

In the given diagram, two unequal circles, centres A and B, intersect at P and Q. The line BP produced meets the circle whose centre is A, at the point R



(a) If $\angle RPQ = x^\circ$, prove that $\angle PBQ = (2x - 180)^\circ$ [4]

$\angle BPQ = 180^\circ - x^\circ$ (angles on st line)

$\therefore \angle PBQ = 180 - 2(180 - x)$ [Δ PBQ isosceles = radii]

$= 180 - 360 + 2x$

$= (2x - 180)^\circ$

(b) Deduce, or prove otherwise, that BQAR is a cyclic quadrilateral [3]

$\angle RAQ$ (reflex) $= 2x$ [angle at centre = 2x angle at circ]

$\therefore \angle RAQ$ (obtuse) $= (360^\circ - 2x)^\circ$

now $360 - \cancel{2x} + \cancel{2x} - 180$ (ie $\angle RAQ + \angle PBQ$)

$= 180^\circ$ (supplementary $\therefore$ BQAR is cyclic).